Theme

"Think deeply about simple things"

STEM Innovation Academy Unit 3

Subject: Integrated Math III Unit Title: Sequences, exponential and logarithmic equations Grade: 10	Teacher: Ahmed Salama Duration: 8 weeks

Summary of Unit

The purpose of this unit is to have full understanding the differences between arithmetic and geometric sequences and the inverse relationship between exponential and logarithmic functions. The lessons and time will be balance among conceptual understanding, procedural practice, and application of these functions.

Stage 1 – Desired Results

Essential Questions:

- Why is a sequence a function?
- How can we use sequences and series to solve real life problems?
- How are geometric sequences and exponential functions connected in theory?
- How are exponential functions and logarithmic functions related?

Enduring Understandings:

- In an arithmetic series, each value in the series differs from its predecessor by the same amount and in geometric series, the ratio of each term to its predecessor is a constant.
- In arithmetic series the function rule is linear and in a geometric series the function rule is exponential
- All arithmetic and geometric sequences can be expressed recursively and explicitly.
- Exponential function is expressed in the form of $f(x) = ab^{x,}$ where b is the constant multiplier and greater than 1, a is the initial value and x is the exponent.
- Exponential value shows rapid growth or rapid decay
- Parent graph of the exponential function has a domain of $(-\infty,\infty)$ and range of $(0,\infty)$, no extrema or symmetry, has an y intercept of (0,1) and the function is increasing.
- Exponential growth model are used mostly for population growth and compound interest.
- Exponential Decay model are used for half-life of chemical compound and also for population that is decreasing.
- Logarithmic function is the inverse of exponential function and has the form of $\log_b y=x$, where b is the base and greater than 1, y is the argument and x is the exponent.

Objectives:

- Recognize arithmetic sequences and geometric sequences.
- Identify the common difference and common ratio
- Recognize sequence is a function and use the explicit expression to find a future term in a sequence
- Use the concept of sequence and series to solve real life problem.
- Derive the Geometric series
- Solve simple exponential equations numerically.
- Apply the laws of exponents in problem solving situation

- Calculate a simple logarithm using the definition.
- Justify properties of logarithms using the definition and properties already developed.
- Use the properties of logarithms to solve problems
- Solve simple logarithmic equations using the definition of logarithm and logarithmic properties.
- Use exponential function or logarithmic functions to model real-life situation and solve the problems.
- Interpret logarithms with irrational values in preparation for graphing logarithmic functions.
- Graph the functions f(x) = log(x), g(x) = log2(x), and h(x) = ln(x) by hand and identify key features of the graphs of logarithmic functions.
- Compare the geometric relationship of the graph of an exponential function to the graph of its corresponding logarithmic function.
- Understand the inverse relationship of logarithm function and the exponential function
- Perform and graph transformations of logarithmic functions.
- Students relate solutions to f(x) = g(x) to the intersection point(s) on the graphs of y = f(x) and y = g(x) in the case where f and g are constant or exponential functions.

Standards/Outcomes: NJSLS

- N.RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})3 = 5^{(1/3)^3}$ to hold, so (51/3)3 must equal 5.

- N.RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

- A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- A.SSE.4 : Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

- F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1

- F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the unction is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and behavior; and periodicity.

- F.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified ,interval. Estimate the rate of change from a graph.*

- F.IF.7.e: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- F-IF.8b: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay

- F.BF.2: Write Arithmetic and Geometric sequence both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

- F-BF.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- F-BF.5: (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents

- F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- F.LE.4: For exponential models, express as a logarithm the solution to abct = d where a, c, and dare numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

- F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context

Unit Math Practice Standards:

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.5** Use appropriate tools strategically.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

Stage 2 – Assessment Evidence

Performance Task(s):
- Performance Task 1: Write arithmetic sequence, find the sum.
- Performance Task 2: Write geometric sequence and find the sum
- Performance Task 4: Solve logarithmic equation.

Authentic Experiences:
- ‘Given arithmetic sequence, find specific term and the sum at that term.
- Given Geometric sequence never end, find the sum to infinite.
- Given exponential equation convert it to logarithmic log. Or natural logarithmic ln.

Unit Pre-Assessment:
- Unit 2 Readiness Assessment
- NWEA Diagnostic Assessment
- Benchmark 2 Assessment

Presentation:
Focus on
- Discuss arithmetic sequences, terms and sum
- Discuss geometric sequences term and sum.
- Discuss the sum of geometric sequences to infinite.
- Present exponential function and solve its equation.
- Convert from exponential functions to logarithmic with base of 10 or other bases.
- Understanding and applying natural functions rules and properties.
- Understanding, investment, and bank interest with different options. (daily, quarterly, annually)

Extensions (Tier I):
- Enrichment Question (challenging add-ons)
- More applications of findings

Differentiation (Tiers 2 and 3):
- Selective grouping
- Extended time
- Small groups / Individual instruction
- Use Bloom’s taxonomy framework to develop multiple-choice, short-answer, matching, and essay question that could lead.
- Use online assessment in our teaching in several ways

Stage 3 – Learning Plan

Vocabulary
- Exponential
- Logarithmic
- Growth
- Decay
- Interest
- Half-life
- A Sequence
- Natural logarithmic
- Base and power
- Product
- Property

- X-intercept/s
- Y-intercept
- Solutions
- Intersects
- Values
- Terms
- Sum
- Coordinates
- Inverse
- Maximum
- Minimum

Learning Materials:
Textbook: CPM Core Connections Algebra 2

Section 5.2.1: How can I undo an exponential function? Finding the Inverse of an Exponential Function (Chapter 5, Online 5-55 – 5-67)
Section 5.2.2: What is a Logarithm? Defining the Inverse of an Exponential Function (Chapter 5, Online 5-68 – 5-80)
Section 5.2.3: What can I learn about logs? Investigating the Family of Logarithmic Functions (Chapter 5, Online 5-81 – 5-92)
Section 5.2.4: How can I transform log functions? Transformations of Logarithmic Functions (Chapter 5, Online 5-93 – 5-104)
Section 6.2.1: How can I solve exponential equations? Using Logarithms to solve Exponential Equations (Chapter 6, Online 6-88 – 6-103)
Section 6.2.2: How can I rewrite it? Investigating the Properties of Logarithms (Chapter 6, Online 6-104 – 6-122)
Section 6.2.3: How can I find an exponential function? Writing Equations of Exponential Functions (Chapter 6, Online 6-123 – 6-136)
Section 6.2.4: Who killed Dr. Dedman? An Application of Logarithms (Chapter 6, Online 6-137 – 6-147)
Section 10.1.1: Can I find a sum without adding? Introduction of arithmetic Series (Chapter 10, Online 10-1 – 10-24)
Section 10.1.2: How can I find a sum without a graph? More Arithmetic Series (Chapter 10, Online 10-25 – 10-33)
Section 10.1.3: How else can I see it? General Arithmetic Series (Chapter 10, Online 10-43 – 10-57)
Section 10.1.4: How else can I express it? Summation Notation and Combinations of Series (Chapter 10, Online 10-58 – 10-70)
Section 10.2.1: What if the series is geometric? Geometric Series (Chapter 10, Online 10-71 – 10-104)
Section 10.2.2: What if n is very large? Infinite Series (Chapter 10, Online 10-105 – 10-112)